Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Pearson

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1. The table below shows corresponding values of x and y for

$$y = 2^{5-\sqrt{x}}$$

The values of y are given to 3 decimal places.

Interval btwn two x-coords = h of trapezium

x	5	5.5	6	6.5	7
y	6.792	6.298	5.858	5.466	5.113

y_1 y_5

multiplied by 2 as all are shared by two trapezia

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_5^7 2^{5-\sqrt{x}}\,dx$$

giving your answer to 2 decimal places.

(3)

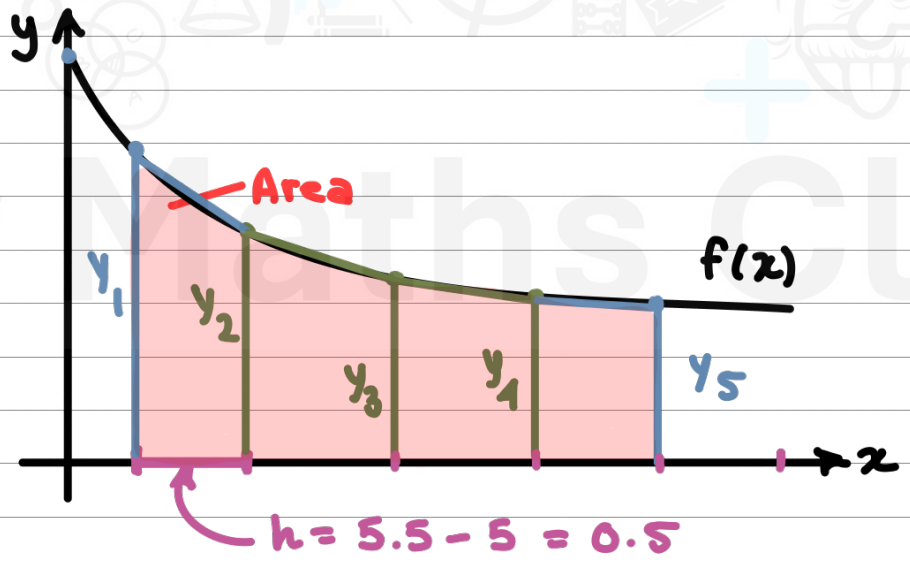
(b) Using your answer to part (a) and making your method clear, estimate

(i) $\int_5^7 2^{6-\sqrt{x}}\,dx$

(ii) $\int_5^7 \left(3 + 2^{5-\sqrt{x}}\right)dx$

(4)

(a)



(Formula is given in formula booklet. The graph is only a visual representation. DO NOT DRAW A GRAPH IN THE EXAM)

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Question 1 continued

$$A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4) + y_5\right]$$

$$A = \frac{1}{2}(0.5)\left(6.792 + 2(6.289 + 5.858 + 5.466) + 5.113\right)$$

$$= \frac{1}{4}(47.131) = 11.782\ldots$$
$$= 11.79 \text{ (2 d.p.)}$$

(b)(i) $\int_5^7 2^{6-\sqrt{x}}\,dx = \int_5^7 2^{1+5-\sqrt{x}}\,dx$

Rewrite using rule : $a^{b+c} = a^b \times a^c$

$$= \int_5^7 2 \times 2^{5-\sqrt{x}}\,dx$$

REMEMBER : $\int ax\,dx = a\int x\,dx$

$$= 2\int_5^7 2^{5-\sqrt{x}}\,dx$$

$$= 2(11.79) = 23.58$$

(ii)

use rule : $\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

$$\int_5^7 3 + 2^{5-\sqrt{x}}\,dx = \int_5^7 3\,dx + \int_5^7 2^{5-\sqrt{x}}\,dx$$

$$= 3x\Big]_5^7 + 11.79$$

$$= 3(7) - 3(5) + 11.79$$

$$= 6 + 11.79 = 23.58$$

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Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 7 marks)

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2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 27x^{\frac{1}{2}} - x^{\frac{3}{2}} - 20 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving each term in simplest form. **(2)**

(b) Hence find the coordinates of the stationary point of C. **(4)**

(c) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of the stationary point of C. **(2)**

(a) $y = 27x^{1/2} - x^{3/2} - 20$

DIFFERENTIATION

$$\boxed{y = ax^n \longrightarrow \frac{dy}{dx} = nax^{n-1}}$$

$$\frac{dy}{dx} = \left(\frac{1}{2}\right)27x^{1/2-1} - \left(\frac{3}{2}\right)x^{3/2-1}$$

$$= \frac{27}{2}x^{-1/2} - \frac{3}{2}x^{1/2}$$

(b) At stationary point, $\frac{dy}{dx} = 0$

$$\frac{27}{2}x^{-1/2} - \frac{3}{2}x^{1/2} = 0$$

$$x^{1/2} \times \left(\frac{27}{2}x^{-1/2} - \frac{3}{2}x^{1/2}\right) = 0 \times x^{1/2}$$

$$\frac{27}{2} - \frac{3}{2}x = 0$$

$$-\frac{2}{3} \times \left(-\frac{3}{2}x\right) = \left(-\frac{27}{2}\right) \times -\frac{2}{3}$$

$$x = 9$$

Leave blank

Question 2 continued

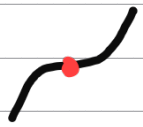
$y = 27(9)^{1/2} - 9^{3/2} - 20 = 34$

so coordinates : $(9, 34)$

(c) RECALL :

if $\frac{d^2y}{dx^2} > 0 \rightarrow$ minimum point

$\frac{d^2y}{dx^2} = 0 \rightarrow$ point of inflection



$\frac{d^2y}{dx^2} < 0 \rightarrow$ maximum point

$$\frac{dy}{dx} = \frac{27}{2}x^{-1/2} - \frac{3}{2}x^{1/2}$$

$$\frac{d^2y}{dx^2} = \left(-\frac{1}{2}\right)\frac{27}{2}x^{-1/2-1} - \left(\frac{1}{2}\right)\frac{3}{2}x^{1/2-1}$$

$$= -\frac{27}{4}x^{-3/2} - \frac{3}{4}x^{-1/2}$$

at $x = 9$:

$$\frac{d^2y}{dx^2} = -\frac{27}{4}(9)^{-3/2} - \frac{3}{4}(9)^{-1/2}$$

$$\frac{d^2y}{dx^2} = -0.5$$

$-0.5 < 0$ so $x = 9$ is a <u>maximum point</u> !

Q2

(Total 8 marks)

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3. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{kx}{4}\right)^8$$

where k is a non-zero constant. Give each term in simplest form.

(4)

$$f(x) = (5 - 3x)\left(2 - \frac{kx}{4}\right)^8$$

In the expansion of $f(x)$, the constant term is 3 times the coefficient of x.

(b) Find the value of k.

(3)

(a) Binomial expansion formula :

$$\boxed{(a+bx)^n = {}^nC_0 a^n (bx)^0 + {}^nC_1 a^{n-1}(bx)^1 + {}^nC_2 a^{n-2}(bx)^2 + \dots}$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n : power of binomial

r : (position of term) − 1

$\left(2 - \frac{kx}{4}\right)^8$ expanded becomes :

$${}^8C_0\, 2^8\left(\frac{-k}{4}x\right)^0 + {}^8C_1\, 2^7\left(\frac{-k}{4}x\right)^1 + {}^8C_2\, 2^6\left(\frac{-k}{4}x\right)^2 + {}^8C_3\, 2^5\left(\frac{-k}{4}x\right)^3$$

${}^8C_0 = 1$ ${}^8C_2 = \frac{8!}{(8-2)!\,2!} = 28$

${}^8C_1 = 8$ ${}^8C_3 = \frac{8!}{(8-3)!\,3!} = 56$

$$= (1)(256) + 8(128)\left(\frac{-k}{4}\right)x + 28(64)\left(\frac{k^2}{16}\right)x^2 + 56(32)\left(\frac{-k^3}{64}\right)x^3$$

$$= 256 - 256kx + 112k^2x^2 - 28k^3x^3$$

(NOTES :- the first and last ${}^nC_r$ are always $=1$
- ${}^nC_1$ is always equal to n)

Leave blank

Question 3 continued

(b) Given in q : constant term = 3 coefficient

$f(x) = (5-3x)(256 - 256kx + 112k^2x^2 - 28k^3x^3)$

coefficient of x $= -3x(256) + 5(-256kx)$
$= (-768 - 1280k)x$

constant term $= 5(256)$
$= 1280$

$1280 = 3(-768 - 1280k)$
$1280 = -2304 - 3840k$
$3584 = -3840k$

$k = \frac{-14}{15}$

Leave blank

Question 3 continued

Leave blank

4. Using the laws of logarithms, solve

$$\log_3(32 - 12x) = 2\log_3(1 - x) + 3$$

(5)

$$\log_3(32-12x) = 2\log_3(1-x) + 3$$

Recall rules of logarithms:

$\log_a(b) + \log_a(c) = \log_a(b \times c)$

$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$

$x\log_a(b) = \log_a(b^x)$

$$\log_3(32-12x) - 2\log_3(1-x) = 3$$

$$\log_3(32-12x) - \log_3(1-x)^2 = 3$$

$$\log_3\left(\frac{32-12x}{(1-x)^2}\right) = 3$$

$$\log_3\left(\frac{32-12x}{1-2x+x^2}\right) = 3$$

RECALL!

$\log_a x = b$

$a^b = x$

$$\frac{32-12x}{1-2x+x^2} = 3^3$$

$$(1-2x+x^2) \times \left(\frac{32-12x}{1-2x+x^2}\right) = (27) \times (1-2x+x^2)$$

$$32-12x = 27 - 54x + 27x^2$$

$$27x^2 - 42x - 5 = 0 \rightarrow \text{FACTORISE}$$

$$(9x+1)(3x-5)$$

$x = -1/9$ or $x = 5/3$ ✗ REJECT!

Leave blank

Question 4 continued

REJECT $x = 5/3$ because :

$\log_3\left(1 - \frac{5}{3}\right) = \log_3(-2/3)$ × undefined !

Q4

(Total 5 marks)

Leave blank

5. $$f(x) = 3x^3 + Ax^2 + Bx - 10$$

where A and B are integers.

Given that

- when $f(x)$ is divided by $(x - 1)$ the remainder is k
- when $f(x)$ is divided by $(x + 1)$ the remainder is $-10k$
- k is a constant

(a) show that

$$11A + 9B = 83$$

(3)

Given also that $(3x - 2)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(3)

(c) Hence find the quadratic expression $g(x)$ such that

$$f(x) = (3x - 2)\,g(x)$$

(2)

(a) USE : REMAINDER THEOREM :

if $\frac{f(x)}{(x-k)}$ = quotient + remainder :

then $f(k)$ = remainder

- if $\frac{f(x)}{(x-1)} = k$ then $f(1) = k$
- if $\frac{f(x)}{(x+1)} = -10k$ then $f(-1) = -10k$

- substitute $x = 1$, $x = -1$ into $f(x)$:

$f(1) = 3(1)^3 + A(1)^2 + B(1) - 10 = k$

$3 + A + B - 10 = k$

$A + B - 7 = k$ (1)

$f(-1) = 3(-1)^3 + A(-1)^2 + B(-1) - 10 = -10k$

$= -3 + A - B - 10 = -10k$

$A - B - 13 = -10k$ (2)

Leave blank

Question 5 continued

SOLVE SIMULTANEOUSLY : + get rid of k!

$[A + B - 7 = k] \times -10$

$A - B - 13 = -10k$

$-10A - 10B + 70 = -10k$

$A - B - 13 = -10k$

$-10A - 10B + 70 = A - B - 13$

$11A + 9B = 83$

(b) Set the factor equal to zero :

$3x - 2 = 0 \longrightarrow x = \frac{2}{3}$

so; $f\left(\frac{2}{3}\right) = 0$

• substitute $x = 2/3$ into $f(x)$:

$3\left(\frac{2}{3}\right)^3 + A\left(\frac{2}{3}\right)^2 + B\left(\frac{2}{3}\right) - 10 = 0$

$\frac{8}{9} + \frac{4}{9}A + \frac{2}{3}B - 10 = 0$

$9 \times \left(\frac{4}{9}A + \frac{2}{3}B - \frac{82}{9}\right) = (0) \times 9$

$4A + 6B - 82 = 0$

$(4A + 6B = 82) \times 3$

$(11A + 9B = 83) \times 2$

$\ominus$ $12A + \cancel{18B} = 246$

$22A + \cancel{18B} = 166$

$-10A = 80$

$A = -8$

$4A + 6B = 82$

$4(-8) + 6B = 82$

$-32 + 6B = 82$

$B = 19$

P 6 6 6 4 6 A 0 1 5 3 6

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Question 5 continued

(c) $f(x) = (3x-2)(g(x))$

Use algebraic division to factorise:

$$3x-2 \overline{\big)\, 3x^3 - 8x^2 + 19x - 10}$$

(1) Divide first term of dividend by first term of divisor: $\frac{3x^3}{3x} = x^2 \rightarrow$ first term of quotient

$$\begin{array}{r|l} & x^2 \\ 3x-2 & 3x^3 - 8x^2 + 19x - 10 \end{array}$$

$\div$

(2) Multiply out and write under:

$$\begin{array}{r|l} & x^2 \\ 3x-2 & 3x^3 - 8x^2 + 19x - 10 \\ & 3x^3 - 2x^2 \end{array}$$

(3) Subtract then bring down next term:

$$\begin{array}{r|l} & x^2 \\ 3x-2 & 3x^3 - 8x^2 + 19x - 10 \\ & -(3x^3 - 2x^2) \downarrow \\ & -6x^2 + 19x \end{array}$$

(4) Divide first term of expression by first term of divisor: $\frac{-6x^2}{3x} = -2x \rightarrow$ next term of quotient

$$\begin{array}{r|l} & x^2 - 2x \\ 3x-2 & 3x^3 - 8x^2 + 19x - 10 \\ & -(3x^3 + 2x^2) \\ & -6x^2 + 19x \end{array}$$

Leave blank

Question 5 continued

(5) Multiply out and write under:

$$
\begin{array}{r|l}
 & x^2 - 2x \\
\hline
3x-2 & 3x^3 - 8x^2 + 19x - 10 \\
 & -(3x^3 + 2x^2) \\
\hline
 & -6x^2 + 19x \\
 & -6x^2 + 4x \\
\hline
\end{array}
$$

(6) Subtract then bring down next term:

$$
\begin{array}{r|l}
 & x^2 - 2x \\
\hline
3x-2 & 3x^3 - 8x^2 + 19x - 10 \\
 & -(3x^3 + 2x^2) \\
\hline
 & -6x^2 + 19x \\
 & -(-6x^2 + 4x) \\
\hline
 & 15x - 10
\end{array}
$$

(7) Divide first term of expression by first term of divisor: $\frac{15x}{3x} = 5$ → next term of quotient

$$
\begin{array}{r|l}
 & x^2 - 2x + 5 \\
\hline
3x-2 & 3x^3 - 8x^2 + 19x - 10 \\
 & -(3x^3 + 2x^2) \\
\hline
 & -6x^2 + 19x \\
 & -(-6x^2 + 4x) \\
\hline
 & 15x - 10
\end{array}
$$

(8) Multiply out and write under:

$$
\begin{array}{r|l}
 & x^2 - 2x + 5 \\
\hline
3x-2 & 3x^3 - 8x^2 + 19x - 10 \\
 & -(3x^3 + 2x^2) \\
\hline
 & -6x^2 + 19x \\
 & -(-6x^2 + 4x) \\
\hline
 & 15x - 10 \\
 & 15x - 10
\end{array}
$$

(Total 8 marks)

Q5

Leave blank

Question 5 continued

(9) Subtract

$$
\begin{array}{r|l}
 & x^2 - 2x + 5 \\
\hline
3x - 2 & 3x^3 - 8x^2 + 19x - 10 \\
 & -(3x^3 + 2x^2) \\
\hline
 & -6x^2 + 19x \\
 & -(-6x^2 + 4x) \\
\hline
 & 15x - 10 \\
 & -(15x - 10) \\
\hline
 & 0
\end{array}
$$

so: $f(x) = (3x-2)\underbrace{(x^2 - 2x + 5)}_{g(x)}$

(Total 7 marks)

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6.

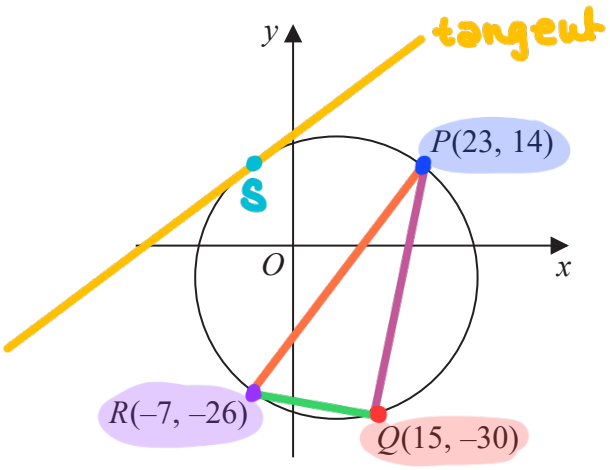


Figure 1

The points $P(23, 14)$, $Q(15, -30)$ and $R(-7, -26)$ lie on the circle C, as shown in Figure 1.

(a) Show that angle $PQR = 90°$

(2)

(b) Hence, or otherwise, find

(i) the centre of C,

(ii) the radius of C.

(3)

Given that the point S lies on C such that the distance QS is greatest,

(c) find an equation of the tangent to C at S, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(a) If $\angle PQR = 90°$, then PQ and QR should be perpendicular.

$m_{PQ} = \frac{14+30}{23-15} = \frac{44}{8} = \frac{11}{2}$

$m_{QR} = \frac{-30+26}{15+7} = \frac{-4}{22} = \frac{-2}{11}$

$m_{PQ} \times m_{QR} = \frac{11}{2} \times \frac{-2}{11} = -1$

so they are inverse reciprocals, hence they are perpendicular.

Leave blank

Question 6 continued

OR : another way :-

If $\measuredangle PQR$ is $90°$, then PQR should be a right-angled triangle with hypotenuse PR

$PQ^2 + QR^2 = PR^2$ (pythagoras theorem)

distance formula : $d = \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$

$(PQ)^2 = (23-15)^2 + (14-(-30))^2$
$= 8^2 + 44^2 = 2000$

$(QR)^2 = (15-(-7))^2 + (-30-(-26))^2$
$= 22^2 + 4^2 = 500$

$(PR)^2 = (23-(-7))^2 + (14-(-26))^2$
$= 30^2 + 40^2 = 2500$

$2000 + 500 = 2500$ which is true, so $\triangle PQR$ is a right-angled triangle, and $\measuredangle PQR$ is $90°$.

(b)(i) Since $PQR = 90°$, according to circle theorems, line PR must be the diameter of the circle. Therefore, the coordinates of the centre are the midpoint :

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{midpoint}_{PR} = \left(\frac{23 + (-7)}{2} \quad \frac{14 + (-26)}{2}\right)$$

$$= (8, -6)$$

So centre $= (8, -6)$

Leave blank

Question 6 continued

(ii) radius $= \dfrac{\text{distance } PR}{2}$

$PR = \sqrt{(23-(-7))^2 + (14-(-26))^2}$

$r = \dfrac{\sqrt{2500}}{2} = 25$

(c) QS is greatest when it is a diameter.

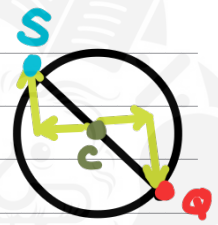
coordinates S :

from C to Q : $x = (15-8) = 7$
$y = (-30-(-6)) = -24$

from C to S ; Subtract 7 from x
add 24 to y

$= ((8-7), (-6+24))$

$= (1, 18)$



so : $m_{radius} = \dfrac{18-(-6)}{1-8} = -\dfrac{24}{7}$

$m_{tangent} = \dfrac{-1}{m_{radius}} = \dfrac{7}{24}$

So : $y = mx + c$
$18 = \dfrac{7}{24}(1) + c \rightarrow c = \dfrac{425}{24}$

$y = \dfrac{7}{24}x + \dfrac{425}{24}$

q asked for form $ax + by + c = 0$;

Leave blank

Question 6 continued

$$\left(y = \frac{7}{24}x + \frac{425}{24}\right) \times 24$$

$$24y = 7x + 425$$

$$7x - 24y + 425 = 0$$

Q6

(Total 8 marks)

Leave blank

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-90° < x < 90°$, the equation

$$3\sin(2x - 15°) = \cos(2x - 15°)$$

giving your answers to one decimal place.

(4)

(ii) Solve, for $0 < \theta < 2\pi$, the equation

$$4\sin^2\theta + 8\cos\theta = 3$$

giving your answers to 3 significant figures.

(4)

(i) $\frac{3\sin(2x-15°)}{3\cos(2x-15°)} = \frac{\cos(2x-15°)}{3\cos(2x-15°)}$

$\frac{\sin(2x-15°)}{\cos(2x-15°)} = \frac{1}{3}$ → $\frac{\sin\theta}{\cos\theta} = \tan\theta$

$\tan(2x-15°) = \frac{1}{3}$

range : $2(-90°)-15 \leq 2x-15 \leq 2(90°)-15$

$-195 \leq 2x-15 \leq 165$

$(2x-15°) = \tan^{-1}(1/3)$

$(2x-15°) = 18.43°$

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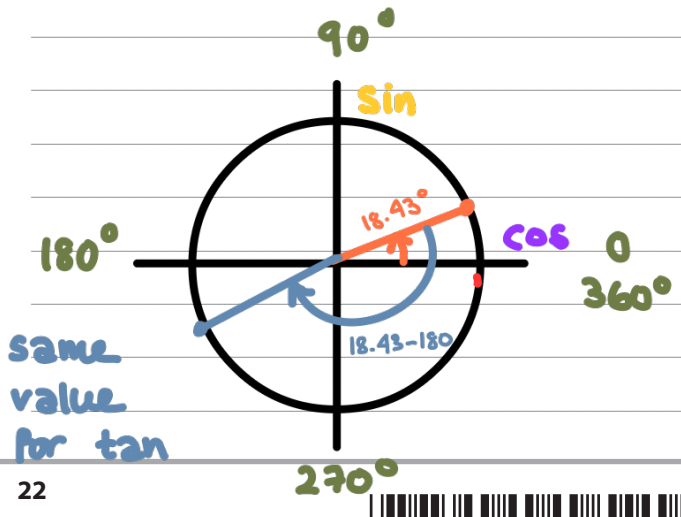


SO:

$(2x-15°) = 18.43$

$(2x-15°) = 18.43 - 180$

$= -161.57$

Leave blank

Question 7 continued

$2x - 15^\circ = 18.43, \; -161.57$
$\quad +15 \qquad +15 \qquad\quad +15$

$\frac{2x}{2} = \frac{33.43}{2}, \; \frac{-146.57}{2}$

$x = 16.7^\circ, \; -73.3^\circ$ (1 d.p.)

(ii) $4\sin^2\theta + 8\cos\theta = 3$

Use identity: $\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$

$4(1-\cos^2\theta) + 8\cos\theta = 3$
$4 - 4\cos^2\theta + 8\cos\theta = 3$

$4\cos^2\theta - 8\cos\theta - 1 = 0$

$\cos\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-1)}}{2(4)}$

$= \frac{8 \pm \sqrt{64 + 16}}{8}$

$\cos\theta = 1 - \frac{\sqrt{5}}{2}$ $\qquad$ $\cos\theta = 1 + \frac{\sqrt{5}}{2}$ ×

reject as range of $y = \cos\theta$ is $-1 < x < 1$

Leave blank

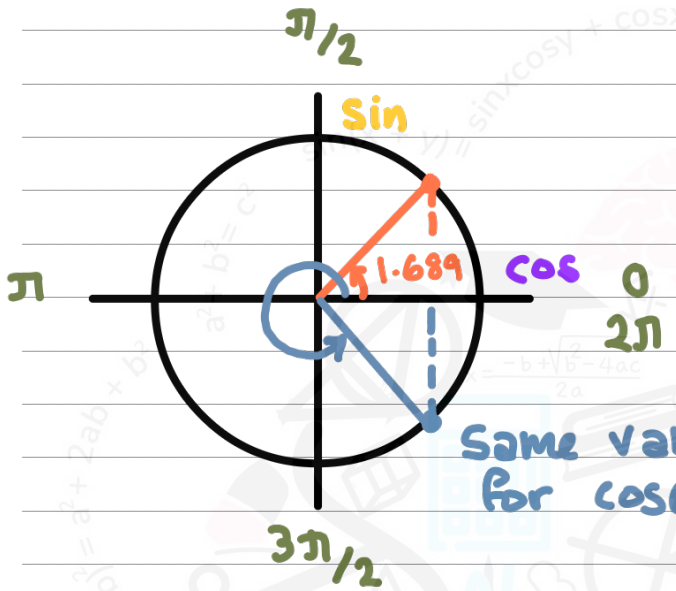
Question 7 continued

$\cos\theta = \frac{1-\sqrt{5}}{2}$, $0 < \theta < 2\pi$

$\theta = \cos^{-1}\left(\frac{1-\sqrt{5}}{2}\right)$

$= 1.689$

ON UNIT CIRCLE :



$\theta = 1.689$

$\theta = 2\pi - 1.689$

$= 4.594$

Same value for $\cos\theta$

$\theta = 1.69$, 4.59 (3 s.f.)

Leave blank

Question 7 continued

(Total 8 marks)

Q7

Leave blank

8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

u_1 • on the 1st hit, the post is driven 100 mm into the ground

u_2 • on the 2nd hit, the post is driven an **additional** 98 mm into the ground

u_3 • on the 3rd hit, the post is driven an **additional** 96 mm into the ground

• the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

partial sum up to u_{20}

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

• the **additional** distances the post travels form a geometric sequence with common ratio r

• on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.

(2)

partial sum up to u_N → 3000 mm

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.

(4)

(a) For an arithmetic series,

100 mm → 98 mm → 96 mm

$d = -2$ $d = -2$

$u_n = a + d(n-1)$

$u_{20} = 100 + (-2)(20-1)$

$= 100 - 38 = 62$

(b) use partial sum formula:

$S_n = \frac{n}{2}\left(2a + d(n-1)\right)$

$S_{20} = \frac{20}{2}\left(2(100) + (-2)(20-1)\right)$

Leave blank

Question 8 continued

$= 10(200 - 38) = 1620$ mm

(c) Now, since we are starting a new series, we treat u_{20} in the arithmetic series as u_1 in the geometric series.

$62 \xrightarrow{r} ? \xrightarrow{r} 60$

u_1 — 20th hit; u_2 — 21st hit; u_3 — 22nd hit

For a geometric series:

$u_n = ar^{n-1}$

$u_3 = u_1 r^{3-1}$

$60 = 62r^2$

$\sqrt{r^2} = \sqrt{\frac{60}{62}} \rightarrow r = 0.983738...$

$= 0.984$ (3 s.f.)

(d) Use partial sum formula:

$S_n = \frac{a_1(1 - r^N)}{(1 - r)}$

[try to use exact calculator value! $= \sqrt{\frac{60}{62}}$]

Keep in mind that:

① We already have the sum of the first 20 terms, $S_{20} = 1620$ mm (part (b))

② hence, a_1 in the sum for the geometric series is the 21st term; u_2.

Calculating $u_2 = ar^{n-1}$

$= 62(0.9837....)^{2-1}$

$= 2\sqrt{930}$ (found using exact value of r)

Leave blank

Question 8 continued

from arithmetic series!

$$S_n = \frac{2\sqrt{930}\,(1-0.9837^n\ldots)}{1-0.9837\ldots} + 1620 > 3000$$

$$\left(\frac{2\sqrt{930}\,(1-0.9837^n\ldots)}{0.01626\ldots} > 1380\right) \times 0.01626\ldots$$

$$\frac{2\sqrt{930}\,(1-0.9837^n\ldots)}{2\sqrt{930}} > \frac{22.4405\ldots}{2\sqrt{930}}$$

$$1-0.9837^n\ldots > 0.3679\ldots$$

$$\left(-0.9837^n\ldots > -0.63207\ldots\right) \times -1$$

$$0.9837^n\ldots < 0.63207\ldots$$

remember: reverse the inequality when multiplying/dividing by a negative number!

$$\log 0.9837^n\ldots < \log 0.63207\ldots$$

$$n \log 0.9837\ldots < \log 0.63207\ldots$$

$$n > \frac{\log 0.63207\ldots}{\log 0.9837\ldots}$$

$\log(0.9837\ldots)$ is negative. reverse the inequality again.

$n > 27.98$

round up to $N = 28$.

$N = 20 + 28 = 48$

Leave blank

Question 8 continued

Q8

(Total 9 marks)

Leave blank

9.

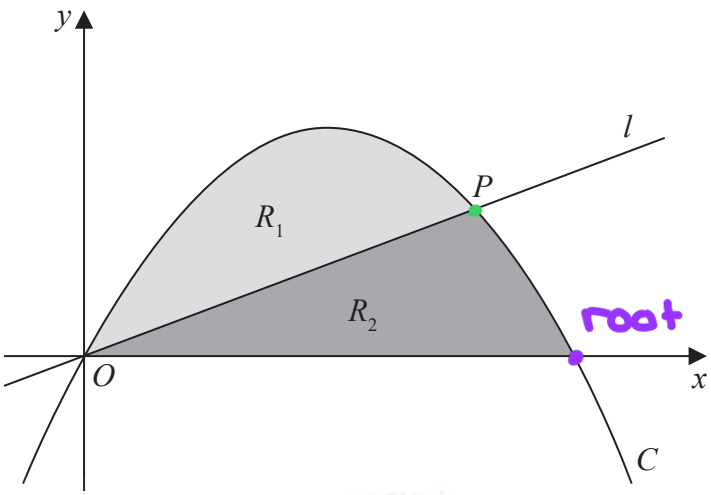


Figure 2

Figure 2 shows

- the curve C with equation $y = x - x^2$
- the line l with equation $y = mx$, where m is a constant and $0 < m < 1$

The line and the curve intersect at the origin O and at the point P.

(a) Find, in terms of m, the coordinates of P.

(2)

The region R_1, shown shaded in Figure 2, is bounded by C and l.

(b) Show that the area of R_1 is

$$\frac{(1-m)^3}{6}$$

(5)

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and l.

Given that the area of R_1 is equal to the area of R_2

(c) find the exact value of m.

(3)

(a) at P, $y_C = y_l$

$x - x^2 = mx$

$\cancel{x}(1-x) = m\cancel{x}$

$m = 1 - x$

$x = 1 - m$

$y = m(1-m)$

Leave blank

Question 9 continued

(b) $A_{R1} = A_{\text{under } C} - A_{\text{under } \ell}$

$$= \int_0^{1-m} x - x^2 \, dx - \int_0^{1-m} mx \, dx$$

REMEMBER!

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1}}$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{1-m} - \left[\frac{m}{2}x^2\right]_0^{1-m}$$

$$= \frac{(1-m)^2 \times 3}{2 \times 3} - \frac{(1-m)^3 \times 2}{3 \times 2} - \frac{m(1-m)^2 \times 3}{2 \times 3} - 0$$

$$= \frac{3(1-m)^2}{6} - \frac{2(1-m)^3}{6} - \frac{3m(1-m)^2}{6}$$

take $3(1-m)^2$ as common factor.

$$= \frac{3(1-m)^2(1-m) - 2(1-m)^3}{6}$$

$$= \frac{3(1-m)^3 - 2(1-m)^3}{6}$$

$$= \frac{(1-m)^3}{6}$$

(c) total shaded area is bounded by the two roots of C:

$$y = x - x^2 = 0$$

$$x(1-x) = 0$$

roots : $x = 0$ and $x = 1$

so $A_{total} = R_2 + R_1 = \int_0^1 x - x^2 \, dx$

Leave blank

Question 9 continued

$$= \left.\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1^2}{2} - \frac{1^3}{3} = \frac{1}{6}$$

if $R_2 = R_1$, then R_1 and R_1 are each equal to $\frac{1}{2}\left(\frac{1}{6}\right) = \frac{1}{12}$

$$R_2 = R_1 = \frac{(1-m)^3}{6}$$

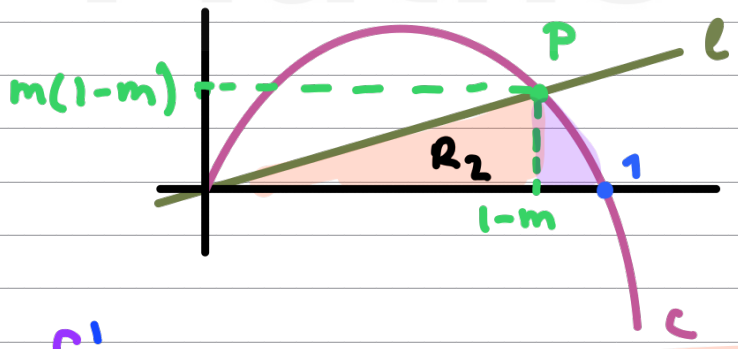
$$\left(\frac{(1-m)^3}{6} = \frac{1}{12}\right) \times 6$$

$$\sqrt[3]{(1-m)^3} = \sqrt[3]{\frac{1}{2}}$$

$$1 - m = \frac{1}{\sqrt[3]{2}}$$

$$m = 1 - \frac{1}{\sqrt[3]{2}}$$

ALTERNATIVE :



$$R_2 = \int_{1-m}^{1} x - x^2 \, dx + \frac{1}{2}(1-m)(m(1-m))$$

Leave blank

Question 9 continued

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{1-m}^{1} + \frac{1}{2}(1-m)(m(1-m))$$

$$= \left[\frac{1^2}{2} - \frac{1^2}{3}\right] - \left[\frac{(1-m)^2}{2} - \frac{(1-m)^3}{3}\right] + \frac{m(1-m)^2}{2}$$

$$= \frac{1}{6} - \frac{(1-m)^2}{2} - \frac{(1-m)^3}{3} + m\frac{(1-m)^2}{2}$$

$$= \frac{1}{6} - (1-m)\frac{(1-m)^2}{2} - \frac{(1-m)^3}{3}$$

$$= \frac{1}{6} - \frac{(1-m)^3 \times 3}{2 \times 3} - \frac{(1-m)^3 \times 2}{3 \times 2}$$

$$= \frac{1}{6} - \frac{(1-m)^3}{6}$$

$$R_2 = \frac{1}{6} - \frac{(1-m)^3}{6} = \frac{(1-m)^3}{6}$$

$$\frac{1}{6} = \frac{(1-m)^3}{6} + \frac{(1-m)^3}{6}$$

$$\frac{1}{\cancel{6}} = \frac{2(1-m)^3}{\cancel{6}}$$

$$\sqrt[3]{(1-m)^3} = \sqrt[3]{\frac{1}{2}}$$

$$1-m = \frac{1}{\sqrt[3]{2}}$$

$$m = 1 - \frac{1}{\sqrt[3]{2}}$$

Q9

(Total 10 marks)

Leave blank

10. (i) Prove by counter example that the statement

"if p is a prime number then $2p + 1$ is also a prime number"

is not true.

(1)

(ii) Use proof by exhaustion to prove that if n is an integer then

$$5n^2 + n + 12$$

is always even.

(4)

(i) try values of p until $2p+1$ is not prime:

$2(7)+1 = 15 = 3(5)$ → not prime!

(ii) If n is even, $n = 2k$
If n is odd, $n = 2k+1$

$5n^2 + n + 12$

even: $5(2k)^2 + 2k + 12$
$= 20k^2 + 2k + 12$
$= 2(10k + k + 6)$ → always even!

odd: $5(2k+1)^2 + 2k + 1 + 12$
$= 5(4k^2 + 4k + 1) + 2k + 1 + 12$
$= 20k^2 + 20k + 5 + 2k + 1 + 12$
$= 20k^2 + 22k + 18$
$= 2(10k^2 + 11k + 9)$ → always even!

both are even so $5n^2 + n + 12$ must be even for all integers n

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 5 marks)

TOTAL FOR PAPER IS 75 MARKS

END